

Name: Mahyar PirayeshDate: Oct. 10th 2022Section 2.2 Quadratic Functions in Standard Form $y = a(x-p)^2 + q$

1. In each of the quadratic function below, indicate the values of the constant "a", "p", "q" and the coordinate of the vertex for a quadratic function in the form of: $y = a(x-p)^2 + q$. Always have AOS in the form of $x = k$

Equation	Constants "a", "p" and "q"	Vertex (Coord)	Equation	Constants "a", "p" and "q"	Vertex (Coord)
a) $y = 2(x-3)^2 + 4$ $y = a(x-p)^2 + q$ Be mindful of $(x-p)^2$ $a(x+5)^2 + q$ $p = -5$ negative	$a = 2$ $p = 3$ $q = 4$	$(x,y) = (3,4)$	g) $y = 9(x+5)^2 + 7$ $= 9$ $= -5$ $= 7$	$a = 9$ $p = -5$ $q = 7$	$(x,y) = (-5,7)$
b) $y = -3(x+0.75)^2 + 6$	$a = -3$ $p = -0.75$ $q = 6$	$(x,y) = (-0.75, 6)$	h) $y = -4x^2 + 10$ $y = -4x^2 + 0x + 10$ $y = -4(x^2 + 0x + \frac{5}{2})$ $y = -4(x)^2 + 10$	$a = -4$ $p = 0$ $q = 10$	$(x,y) = (0,10)$
c) $y = -\frac{2}{3}(x-1)^2 - 2$	$a = -\frac{2}{3}$ $p = 1$ $q = -2$	$(x,y) = (1, -2)$	i) $y = -3(x + \frac{2}{3})^2 - 2$	$a = -3$ $p = -\frac{2}{3}$ $q = -2$	$(x,y) = (-\frac{2}{3}, -2)$
d) $y = (-3x)^2 + 2$ $y = 9(x)^2 + 2$	$a = 9$ $p = 0$ $q = 2$	$(x,y) = (0, 2)$	j) $y = \frac{3(x-11)^2 + 4}{2}$ $y = \frac{3}{2}(x-11)^2 + 2$	$a = \frac{3}{2}$ $p = 11$ $q = 2$	$(x,y) = (11, 2)$
e) $y = (2x-1)^2 - 3$ $y = 4x^2 - 4x + 1 - 3$ $y = 4(x^2 - x - \frac{1}{2})$ $y = 4(x - \frac{1}{2})^2 - 3$	$a = 4$ $p = \frac{1}{2}$ $q = -3$	$(x,y) = (\frac{1}{2}, -3)$	k) $y = (3x+1)^2 + 8$ $y = (3(x + \frac{1}{3}))^2 + 8$ $y = 9(x + \frac{1}{3})^2 + 8$	$a = 9$ $p = -\frac{1}{3}$ $q = 8$	$(x,y) = (-\frac{1}{3}, 8)$

$\Downarrow$
 $y = (2(x - \frac{1}{2}))^2 - 3$
 $y = 4(x - \frac{1}{2})^2 - 3$

$f) y = (3x-6)^2 - 8$ $y = [3(x-2)]^2 - 8$ $y = 9(x-2)^2 - 8$	$a=9$ $p=-2$ $q=-8$	(x,y) $=(-2, -8)$	$l) y = 4x^2 + 4x + 9$ $y = 4(x^2 + x + \frac{9}{4})$ $y = 4(x + \frac{1}{2})^2 + 8$	$a=4$ $p=-\frac{1}{2}$ $q=8$	(x,y) $=(-\frac{1}{2}, 8)$
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2. Factor each of the following quadratic functions and find i) the Coordinates of the Roots, ii) the Equation of the Axis of Symmetry, iii) Coordinates of the Vertex, iii) Domain and Range

a) $y = x^2 - 5$ Vertex: Roots: $y = (x)^2 - 5$ $a=0$ $p=0$ $q=-5$ $x^2 - 5 = 0$ $x = \pm \sqrt{5}$ $x = 0$ $AOS: p = 0$ $x = 0$ Range: $y \geq -5$ Roots: $x_1 = \sqrt{5}$ $x_2 = -\sqrt{5}$ A of S: $x = 0$ Vertex: $(0, -5)$ Range: $y \geq -5$	b) $y = -2(x+2)^2$ Vertex: Roots: $-2(x+2)^2 = 0$ $a=-2$ $p=2$ $q=0$ $(x,y) = (-2, 0)$ $AOS: x = p = -2$ Range: $y \leq 0$ Roots: $x = -2$ A of S: $x = -2$ Vertex: $(-2, 0)$ Range: $y \leq 0$	c) $y = 5(x-5)^2 - 10$ Vertex: Roots: $5(x-5)^2 - 10 = 0$ $p=5$ $q=-10$ $(x,y) = (5, -10)$ $AOS: x = p = 5$ Range: $y \geq -10$ Roots: $x_1 = 5 + \sqrt{2}$ $x_2 = 5 - \sqrt{2}$ A of S: $x = 5$ Vertex: $(5, -10)$ Range: $y \geq -10$
d) $y = 7x^2 - 14$ Vertex: Roots: $y = 7(x)^2 - 14$ $p=0$ $q=-14$ $(x,y) = (0, -14)$ $AOS: x = p = 0$ Range: $y \geq -14$ Roots: $x = \pm \sqrt{2}$ Roots: $\pm \sqrt{2}$ A of S: $x = 0$ Vertex: $(0, -14)$ Range: $y \geq -14$	e) $y = (4x-4)^2 - 10$ Vertex: Roots: $y = 16(x-1)^2 - 10$ $p=1$ $q=-10$ $(x,y) = (1, -10)$ $AOS: x = p = 1$ Range: $y \geq -10$ Roots: $x = 1 \pm \frac{\sqrt{10}}{2}$ Roots: $1 \pm \frac{\sqrt{10}}{2}$ A of S: $x = 1$ Vertex: $(1, -10)$ Range: $y \geq -10$	f) $y = 5(3x)^2$ Vertex: Roots: $y = 45(x)^2$ $p=0$ $q=0$ $(x,y) = (0, 0)$ $AOS: x = p = 0$ Range: $y \geq 0$ Roots: $x = 0$ A of S: $x = 0$ Vertex: $(0, 0)$ Range: $y \geq 0$
g) $y = \frac{(5x-5)^2 + 15}{5}$ Roots: $y = \frac{25(x-1)^2 + 15}{5} = 5(x-1)^2 + 3$ Vertex: Roots: $5(x-1)^2 + 3 = 0$ $p=1$ $q=3$ $(x,y) = (1, 3)$ $x = 1 \pm \frac{\sqrt{3}}{5}$ No roots Range: $y \geq 3$ Roots: $\emptyset$ A of S: $x = 1$ Vertex: $(1, 3)$ Range: $y \geq 3$	h) $y = -2(3-x)^2 - 14$ Roots: $y = -2[-(x-3)]^2 - 14$ Vertex: Roots: $y = -2(x-3)^2 - 14$ $p=3$ $q=-14$ $(x,y) = (3, -14)$ $x = 3 \pm \sqrt{7}$ No roots Range: $y \leq -14$ Roots: $\emptyset$ A of S: $x = 3$ Vertex: $(3, -14)$ Range: $y \leq -14$	i) $y = \frac{2\sqrt{(x^2+8x^2+16)} + 4}{2} - 10$ Roots: $y = \frac{2(x^2+4)}{2} + \frac{4}{2} - 10$ Vertex: Roots: $y = x^2 - 4$ $p=0$ $q=-4$ $(x,y) = (0, -4)$ $x = \pm 2$ Range: $y \geq -4$ Roots: $x = \pm 2$ A of S: $x = 0$ Vertex: $(0, -4)$ Range: $y \geq -4$

3. Graph the following quadratic functions and label the Roots, Axis of Symmetry, Vertex, and Y-intercepts:

a) $y = (x-4)^2 - 9$

Roots:

$$(x-4)^2 - 9 = 0$$

$$x-4 = \pm 3$$

$$x_1 = 7 \quad x_2 = 1$$

y-int:

$$y = (0-4)^2 - 9 = 7$$

Vertex:

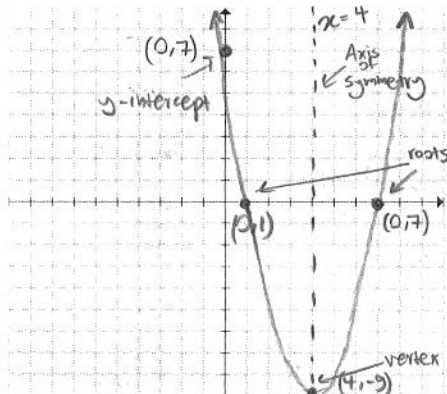
$$p = 4$$

$$q = -9$$

$$(x,y) = (4, -9)$$

AOS:

$$x = p = 4$$



b) $y = -3(x+2)^2 + 8$

Roots:

$$-3(x+2)^2 + 8 = 0$$

$$x+2 = \pm \sqrt{\frac{8}{3}}$$

$$x = \pm 2\sqrt{\frac{2}{3}} - 2$$

$$x_1 = \frac{2\sqrt{6}}{3} - 2 \approx -0.37$$

$$x_2 = -\frac{2\sqrt{6}}{3} - 2 \approx -3.63$$

y-int:

$$y = -3(2)^2 + 8 = -4$$

Vertex:

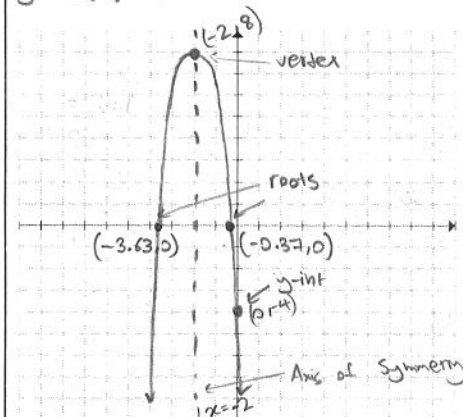
$$p = -2$$

$$q = 8$$

$$(x,y) = (-2, 8)$$

AOS:

$$x = p = -2$$



b) $y = \frac{1}{3}(x+3)^2 + 1$

Roots:

$$\frac{1}{3}(x+3)^2 + 1 = 0$$

$$x+3 = \pm \sqrt{-3}$$

NO ROOTS

y-int:

$$y = \frac{1}{3}(3)^2 + 1 = 4$$

Vertex:

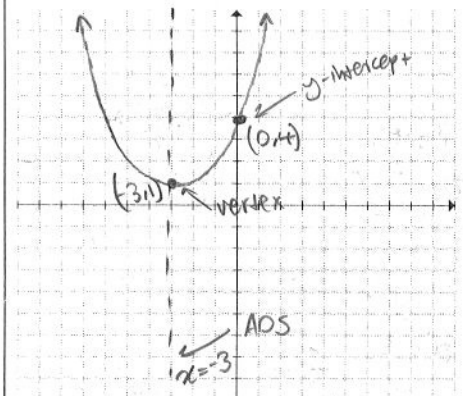
$$p = -3$$

$$q = 1$$

$$(x,y) = (-3, 1)$$

AOS:

$$x = p = -3$$



c) $y = -\frac{1}{2}(x-2)^2 + 7$

Roots:

$$-\frac{1}{2}(x-2)^2 + 7 = 0$$

$$(x-2)^2 = 14$$

$$x = 2 \pm \sqrt{14}$$

$$x_1 \approx 5.74$$

$$x_2 \approx -1.74$$

y-intercept:

$$y = -\frac{1}{2}(-2)^2 + 7 = 5$$

Vertex:

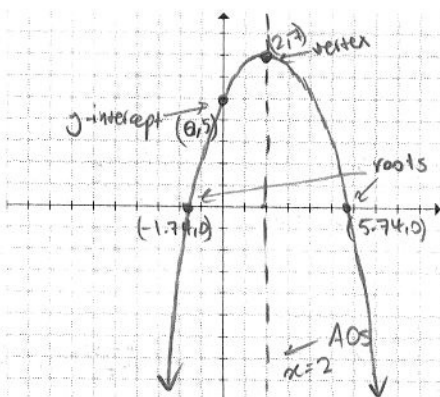
$$p = 2$$

$$q = 7$$

$$(x,y) = (2, 7)$$

AOS:

$$x = p = 2$$



d) $y = \frac{(2x-4)^2 + 8}{8}$

$$y = \frac{4(x-2)^2}{8} + 1 = \frac{1}{2}(x-2)^2 + 1$$

Roots:

$$\frac{1}{2}(x-2)^2 + 1 = 0$$

$$x-2 = \pm \sqrt{-2}$$

No roots

y-intercept:

$$y = \frac{1}{2}(-2)^2 + 1 = 3$$

Vertex:

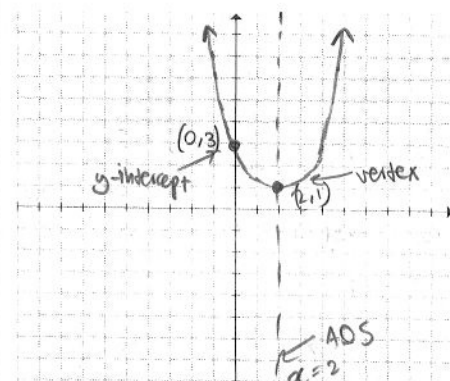
$$p = 2$$

$$q = 1$$

$$(x,y) = (2, 1)$$

AOS:

$$x = p = 2$$



e) $y = (3x-15)^2 - 6$

$$y = 9(x-5)^2 - 6$$

Roots:

$$9(x-5)^2 - 6 = 0$$

$$x-5 = \pm \sqrt{\frac{2}{3}}$$

$$x = 5 \pm \sqrt{\frac{2}{3}}$$

$$x_1 \approx 5.82$$

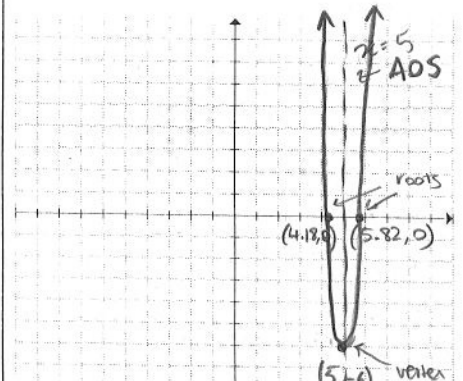
$$x_2 \approx 4.18$$

Vertex:

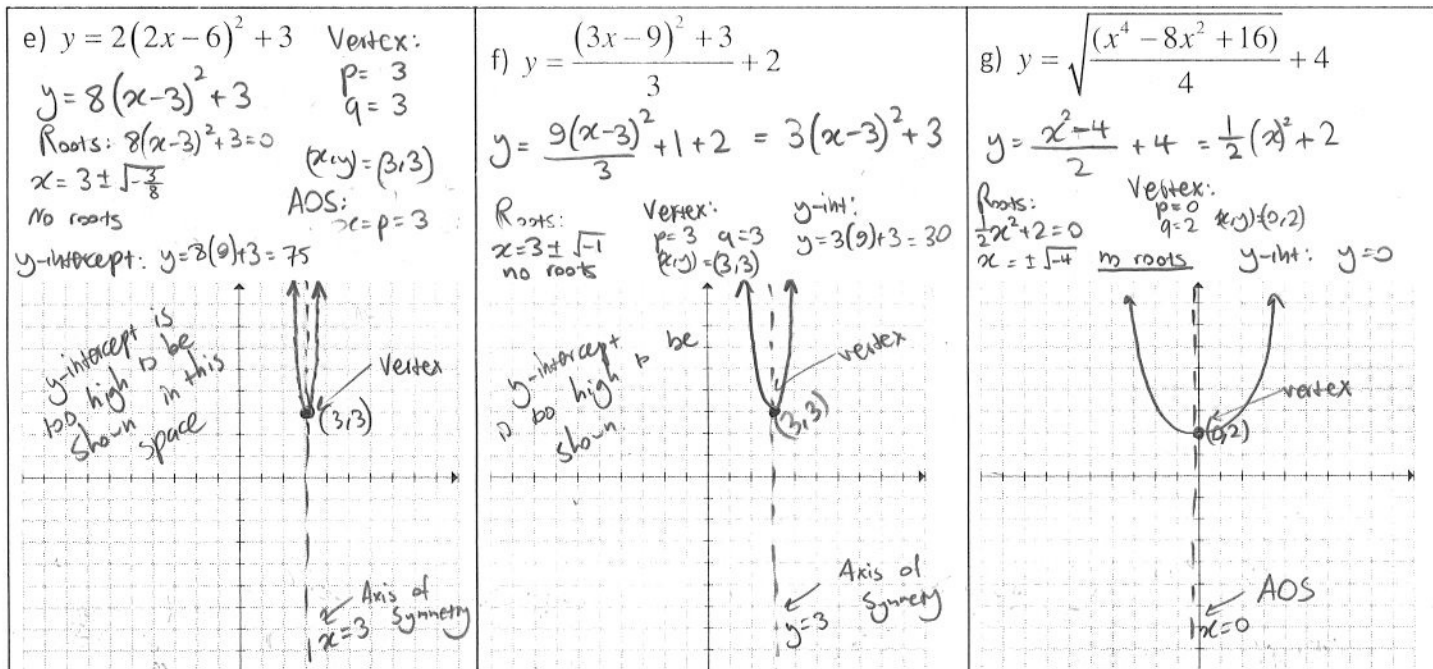
$$p = 5$$

$$q = -6$$

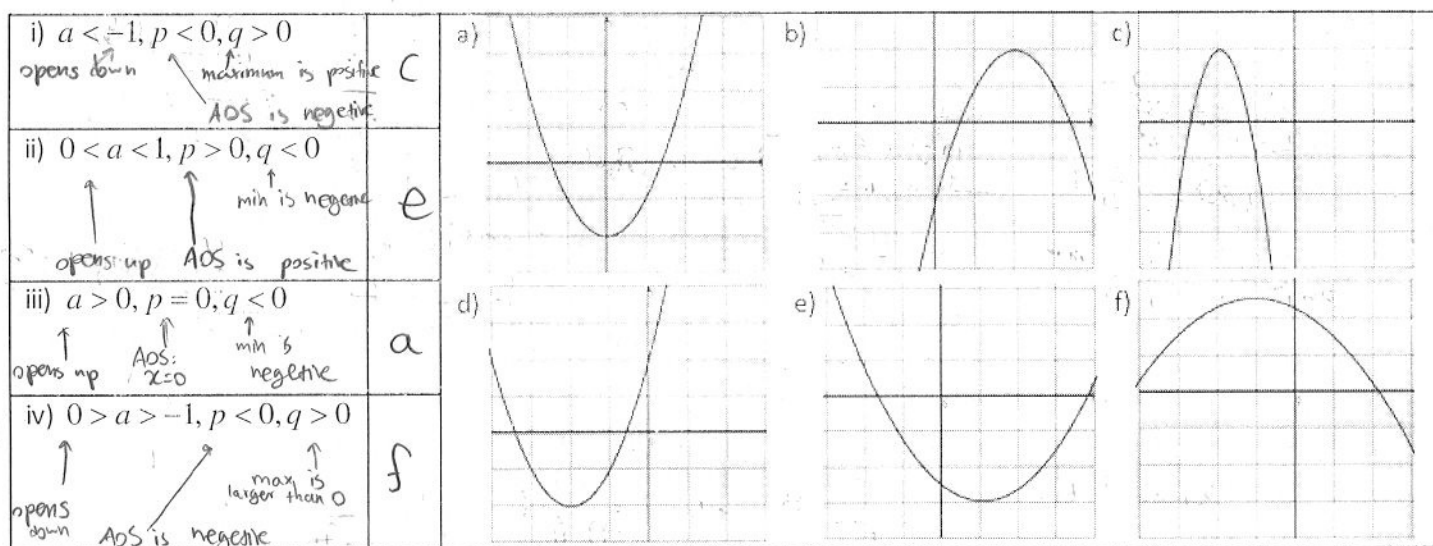
$$(x,y) = (5, -6)$$



the graph. the graph will be "fat".
 α is small, so graph will be "fat".
 the larger " α " is, the skinnier the graph.



4. If each parabola is in the form of $y = a(x-p)^2 + q$, then which graph best describes each equation:



5. Convert the function $y = \frac{1}{2}x^2 - 4x + 1$ into standard form.

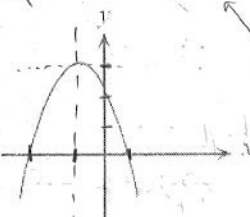
$$y = \frac{1}{2}x^2 - 4x + 1 \quad \left\{ \begin{array}{l} y = \frac{1}{2}(x-4)^2 + 7 \\ y = \frac{1}{2}(x^2 - 8x + 2) \end{array} \right.$$

$$\begin{aligned} y &= a(x-p)^2 + q \\ y &= \frac{1}{2}(x-4)^2 + 7 \\ a &= \frac{1}{2} \quad p=4 \quad q=7 \end{aligned}$$

6. The graph of the function $y = ax^2 + bx + c$ is shown in the diagram. Which of the following statements below must be positive?

- a) a b) bc c) ab^2 d) $b-c$

"a" must be negative since graph opens down



"b" must be negative since the graph has been shifted up.

since graph is shifted to the left

"c" is positive
 "a" is negative

Roots:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{aligned} y &= a(x-p)^2 + q \\ y &= ax^2 - 2apx + ap^2 + q \\ \text{vertex: } (p,q) \\ c &= ap^2 + q \\ |p| &< |q| \end{aligned}$$

7. Consider the parabola $y = 5x^2 - 4x + c$. The value of the real number "c" for which such a parabola touches the x-axis exactly once is: [BCSSM 2008]

a) $-\frac{4}{5}$ b) 0 c) $\frac{2}{5}$ **(d) $\frac{4}{5}$** e) $\frac{\sqrt{5}}{4}$

$y = ax^2 + bx + c$
 $a = 5$
 $b = -4$

$\Delta = (-4)^2 - 4(5)(c) = 0$

$16 - 20c = 0$

$16 - 20c = 0$

$c = \frac{4}{5}$

for the parabola to touch the x-axis once, the discriminant in $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ must be equal to 0.

stop mark

8. Point "A" and "B" are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of $\overline{AB}$. What is the length of $\overline{AB}$?

$A = (x_1, y_1)$

$B = (x_2, y_2)$

$y_1 = 4x_1^2 + 7x_1 - 1$

$y_2 = 4x_2^2 + 7x_2 - 1$

$\frac{x_1 + x_2}{2} = 0$

$\frac{y_1 + y_2}{2} = 0$

$x_1 + x_2 = 0$

$x_1 + x_2 = 0$

$y_1 + y_2 = 0$

$4x_1^2 + 7x_1 - 1 + 4x_2^2 + 7x_2 - 1 = 0$ because x_1 & x_2 are interchangeable in the equation

9. The parabola $y = x^2 - 2x + 4$ is moved "p" units to the right and "q" units down. The x-intercepts of the resulting parabola are 3 and 5. What are the values of "p" and "q"?

$y = x^2 - 2x + 4$

$y = x^2 - 2x + 1 + 3$

$y = (x - 1)^2 + 3$

$y = a(x - p)^2 + q$

vertex = (p, q) = (1, 3)

since the whole graph has shifted, the vertex has also shifted.

New graph's equation:

$y = (x - (1 + p))^2 + 3 - q$

lets plug in (3, 0) & (5, 0)

$0 = (2 - p)^2 + 3 + q = p^2 - 4p + q + 7$

$0 = (4 - p)^2 + 3 - q$

$0 = p^2 - 8p - q + 19$

subtract the equations

$-4p + 12 = 0$

$4p = 12$

$p = 3$

$0 = (3)^2 - 8(3) - q + 19$

$0 = 4 - q$ $q = 4$

$p = 3$
 $q = 4$

length

$\frac{1}{2} - (-\frac{1}{2}) = 1$

height

$\frac{7}{2} - (-\frac{7}{2}) = 7$

length: $5\sqrt{2}$ units

$5\sqrt{2}$

1

stop mark

10. If $y = a(x - 2)^2 + c$ and $y = (2x - 5)(x - b)$ represents the same quadratic function, what is the value of the constant "b"?

$y = a(x - 2)^2 + c$

$y = a(x - p)^2 + q$

ADS $p = x$

ADS $x = 2$

$y = 2x^2 + (-2b - 5)x + 5b$

ADS: $x = \frac{-b}{2a} = \frac{-2b - 5}{4}$

$2 = \frac{-2b - 5}{4}$

$2b = 8 - 5 = 3$

$b = \frac{3}{2}$

Redo

11. The parabola $y = ax^2 + bx + c$ has vertex (p, p) and y-intercept (0, -p), where $p \neq 0$, what is the value of "b"?

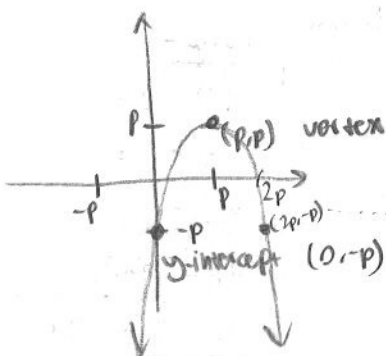
a) -p

b) 0

c) 2

(d) 4

e) p



$-p = a(0)^2 + b(0) + c$

$p = ap^2 + bp - p$

$-p = c$

$ap^2 + (b - 2)p = 0$

$p(ap + b - 2) = 0$

$ap + b - 2 = 0$

$b = -ap + 2$

$a = \frac{2 - b}{p}$

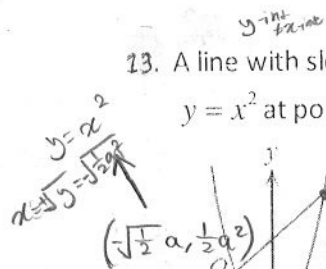
factor p^2 $8p^2 - 4p^2b + 2bp^2 = 0$
 $8 - 4b + 2b = 0$
 $b = 4$

12. The parabola $y = x^2 - 2x + 4$ is moved 'p' units to the right and 'q' units down. The x-intercepts of the resulting parabola are 3 and 5. What are the values of 'p' and 'q'?

Same question as #9 on the previous page.

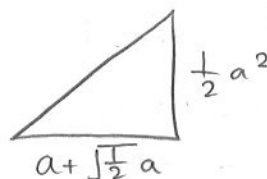
$$\boxed{\begin{matrix} p=3 \\ q=4 \end{matrix}}$$

13. A line with slope 1 passes through the point "P" on the negative x-axis as shown and intersects the parabola $y = x^2$ at points Q and R. If $PQ = RQ$, then what is the y-intercept of line PR?



$$y = x^2$$

$$y = (a)^2 = a^2$$



$$\text{rise} = \text{run}$$

$$a + \sqrt{\frac{1}{2}}a = \frac{1}{2}a^2$$

$$a^2 - 2a - 2\sqrt{\frac{1}{2}}a = 0$$

$$a^2 - 2a(1 + \sqrt{\frac{1}{2}}) = 0$$

$$a[a - 2(1 + \sqrt{\frac{1}{2}})] = 0$$

$$a_1 = 0 \quad a_2 = 2 + 2\sqrt{\frac{1}{2}}$$

$$y = x + b$$

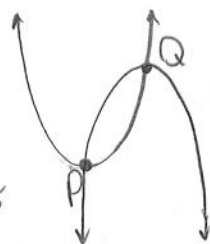
$$(x, y) = (2 + 2\sqrt{\frac{1}{2}}, 4 + 8\sqrt{\frac{1}{2}} + 2)$$

$$4 + 8\sqrt{\frac{1}{2}} + 2 = 2 + 2\sqrt{\frac{1}{2}} + b$$

$$\boxed{\text{y-intercept of PR} = b = 6\sqrt{\frac{1}{2}} + 4 = 3\sqrt{2} + 4}$$

14. Challenge: The parabola $y = f(x) = x^2 + bx + c$ has vertex "P" and the parabola $y = g(x) = -x^2 + dx + e$ has vertex "Q", where "P" and "Q" are distinct points. The two parabolas also intersect at "P" and "Q".

- i) Prove that $2(e - c) = bd$.



$$y = a(x - p)^2 + q$$

$$\text{vertex} = (x, y) = (p, q)$$

$$P: \text{vertex of } f(x) = \left(-\frac{b}{2}, c - \frac{b^2}{4}\right)$$

$$Q: \text{vertex of } g(x) = \left(\frac{d}{2}, e + \frac{d^2}{4}\right)$$

$$\text{Since the point } Q = \left(\frac{d}{2}, e + \frac{d^2}{4}\right) \text{ is on } f(x), \text{ we can plug it in.}$$

- ii) Prove that the line through points "P" and "Q" has slope $\frac{1}{2}(b + d)$ and y-intercept $\frac{1}{2}(c + e)$.

$$\text{Slope of } \overline{PQ}: \frac{(x_2, y_2) - (x_1, y_1)}{x_2 - x_1}$$

$$\frac{c - \frac{b^2}{4} - \left(e + \frac{d^2}{4}\right)}{-\frac{b}{2} - \frac{d}{2}} = \frac{c - \frac{b^2}{4} - e - \frac{d^2}{4}}{-\frac{b + d}{2}} = \frac{4c - b^2 - 4e - d^2}{-b - d} = \frac{b^2 + d^2 + 4(e - c)}{2(b + d)}$$

$$= \frac{b^2 + 2bd + d^2}{2(b + d)} = \frac{(b + d)^2}{2(b + d)} = \frac{b + d}{2} = \frac{1}{2}(b + d)$$

$$\boxed{\text{slope} = \frac{1}{2}(b + d)}$$

$$y = \frac{b + d}{2}x + h$$

$$\text{y-intercept} = h = \frac{1}{2}(c + e)$$

$$bd = 2e - 2c$$

$$h = 4e - 2e + 2c$$

$$\boxed{n = 2e + 2c = 2(e + c)}$$

$$y = \frac{1}{2}(b + d)x + 2(e + c)$$